AFCRL -72-0207

HYDROMAGNETIC WAVE PHENOMENA

IN THE MAGNETOSPHERE

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Raleigh, North Carolina

Contract No. F19628-67-C0152

Project   8601

Task   860.07

Work Unit No.   86010701

FINAL REPORT

Covering Period 2 January 1967 - 31 December 1970

Contract Monitor: Elwood Maple
Space Physics Laboratory

1 February 1971

Approved for public release; distribution unlimited.
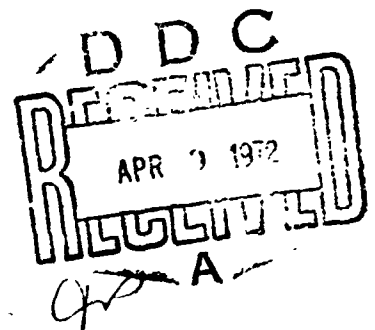
Prepared For

Air Force Cambridge Research Laboratories

Air Force Systems Command

United States Air Force

Bedford, Massachusetts 01730

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
North Carolina State University Raleigh, North Carolina 27607	Unclassified
	2b. GROUP

3. REPORT TITLE

HYDROMAGNETIC WAVE PHENOMENA IN THE MAGNETOSPHERE

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Scientific.   Final.   2 January 1967 - 31 December 1970

5. AUTHOR(S) *(First name, middle initial, last name)*

Alvin W. Jenkins, Jr.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1 February 1971	31	15

8a. CONTRACT OR GRANT NO. F19628-67-C-0152	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT, TASK, WORK UNIT NOS. 8601-07-01	
c. DOD ELEMENT 61102F	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d. DOD SUBELEMENT 681311	AFCRL-72-0207

10. DISTRIBUTION STATEMENT

A - Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
TECH, OTHER	Air Force Cambridge Research Laboratories (PH) L. G. Hanscom Field Bedford, Massachusetts 01730

13 ABSTRACT

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    A third model utilizes a curved, dipole-like magnetic field. However, computations are not complete for this case.

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The propagation characteristics for three model magnetospheres are compared for hydromagnetic waves in the micropulsation frequency range, 10^{-4} rad/sec $< \omega < 1.0$ rad/sec. One model is of a plane, layered medium over a plane, conducting earth. A second model is one composed of concentric cylindrical shells of plasma surrounding a conducting, cylindrical earth. In the plane model the magnetic field is horizontal, but varies with height. In the second, the static magnetic field is parallel to the cylindrical axis, but again varies with height. The transmission coefficient for the amplitude of the magnetic field of an incident plane hydromagnetic wave is computed as a function of frequency for both models, and the results are compared. The scattering and diffraction effects present in the model with curved surfaces substantially reduce the transmission coefficient at higher frequencies. Also, differences in geometric shape are much more significant than any resonable variation of other magnetospheric parameters within either model.

A third model utilizes a curved, dipole-like magnetic field. However, computations are not complete for this case.

I. INTRODUCTION

The objective of the research supported by Air Force Contract F19628-67-C0152 was the investigation of hydromagnetic phenomena in the magnetosphere in order to determine the possible physical mechanisms for the production of the observed geomagnetic micropulsations whose energies are confined to comparatively narrow frequency bandwidths. Essentially all the work performed was directed towards understanding the propagation characterics of the magnetosphere.

This work was begun January 2, 1967 and concluded with the termination of the research contract on December 31, 1970.

The work described in this report is primarily the theoretical determination of the propagation characteristics of low frequency hydromagnetic waves in three model magnetospheres: (1) a plane, layered medium with horizontal magnetic field; (2) a cylindrical, layered medium with static magnetic field parallel to the cylindrical axis; (3) a cylindrical model with static field normal to the cylindrical axis.

II. MAGNETOSPHERIC PROPAGATION OF HYDROMAGNETIC WAVES

A. Introduction. A large portion of the work covered by this report was published in a paper "A Comparison of Hydromagnetic Wave Propagation in Magnetospheres of Plane and Cylindrical Geometries," *Journal of Geophysical Research*, Vol. 75, pp. 3817-3824, 1970. Portions of the material in sections A, B, and C of this chapter are quoted from this paper

The mathematical description of the propagation of hydromagnetic waves through the magnetosphere is complicated by many factors: (1) the medium is inhomogeneous, its properties varying by many orders of magnitude over distances less than a wavelength; (2) the static magnetic field surrounding the earth causes the medium to be anisotropic; (3) the magnetospheric medium is dissipative; (4) the parameters that characterize the medium are not known experimentally, except approximately; (5) the geometry of the magnetosphere does not lend itself to a simple mathematical formulation of the propagation problem. Because of these complications, a realistic magnetospheric model has never been used in propagation calculations at hydromagnetic wave frequencies. A number of workers have, however, reported calculated result based on approximate models [e.g., Field and Greifinger, 1965, 1966; Francis and Karplus, 1960; Greifinger and Greifinger, 1965; Jacobs and Watanabe, 1962, Karplus et al., 1962; Prince and Bostick, 1964; Prince et al., 1964].

A typical treatment takes into account some of the factors listed above. A 'full wave' solution to the propagation equations can be affected numerically, even for the inhomogeneous, anisotropic,

dissipative medium, if the magnetospheric parameters are functions of only one Cartesian coordinate. In such an analysis the earth becomes an infinite half-space with a plane boundary, the magnetosphere is layered slab, and the fields everywhere are superpositions of plane waves By allowing the parameters of the medium to vary with height in a sufficiently simple fashion, some workers have obtained analytic expressions for transmission coefficients [Field and Greifinger, 1965, 1966; Greifinger and Greifinger, 1965]. In general, the tendency has been to try coping with the first four difficulties listed previously, but to restrict the geometry of the model. Some work has been done on resonant modes in a spherically symmetric magnetosphere [Radoski and Corovillano, 1966; Corovillano et al., 1965]. No use seems to have been made of these results in predicting transmission characteristics, however. In general, the analysis of magnetospheric propagation of hydromagnetic waves has been one dimensional and has made use of plane waves and plane bounding surfaces. A magnetospheric model that includes curved surfaces would also include scattering and diffraction effects absent in the plane layered models.

Thus it is reasonable to attempt a solution to the propagation problem for a curved (though highly artificial) model in order to evaluate the effects of curvature and hence the suitability or accuracy of the plane laminar models.

Three calculations have been attempted; two have been completed. The three cases are:

1. A calculation leading to a solution to the problem of the propagation of a hydromagnetic disturbance through a cylindrically

symmetric magnetosphere of a plane wave incident upon it. The

continuously variable nature of the medium is approximated

by a number of homogeneous cylindrical shells of plasma. The

'dipole' field of the earth in this model varies only with

distance from the cylindrical axis and is parallel to the axis.

A plane normal to the axis of this model is thus comparable to

the equatorial plane.

2. A second calculation is based on a plane model of the magneto-

sphere. In this simpler case, the same parameter profiles are

used as in the cylindrical model, and plane layers have the

same thickness and height as the corresponding cylindrical shells.

When the transmission coefficients in the two cases are compared,

the effects of curvature of the magnetospheric surfaces are

revealed.

The exact results are obtained by assuming zero collision

frequencies throughout the magnetosphere in both cases. However,

the effect of molecular collisions on the propagation is accounted

for approximately by making use of the ionospheric transmission

coefficient computed by Field and Greifinger [1966] to modify

the exact no-collision results. Although this composite treatment

is not a rigorous solution to the propagation equations with

collisions, it is thought to be a reasonable approximation to the

problem. In addition, this approximate solution requires con-

siderably less computing time.

3. A third propagation calculation was attempted on a model

magnetosphere which again was cylindrically symmetric. However,

in this third case the magnetic field was chosen to lie in the

plane normal to the cylinder axis and to exhibit a "dipole"

characteristic. Thus the direction and magnitude of the static
field were functions of position, varying, in fact, with both r
and ϕ, the radius and azimuthal angle in cylindrical coordinates.
This considerably more complicated magnetic field structure
leads of course to a much more difficult mathematical problem.
At the date of termination of this contract the solution to
the propagation problem for this third case was incomplete.
A description of the progress made, however, will be found in
section C below.

The analysis of the propagation problem for the first two
cases proceeds as follows.

Consider a homogeneous plasma medium, corresponding to any
one of th. layers in the magnetospheric model. Here, Maxwell's
equations (mks units) take the form

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \qquad \nabla \times \vec{E} = -\mu_0 \vec{H}\partial/\partial t$$

$$\nabla \cdot \vec{H} = 0 \qquad \nabla \times \vec{H} = \epsilon_0(\partial \vec{E}/\partial t) + \vec{J} \tag{1}$$

For time dependence of the form $e^{i\omega t}$, these equations become

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \qquad \nabla \times \vec{E} = -i\mu_0\omega\vec{H}$$

$$\nabla \cdot \vec{H} = 0 \qquad \nabla \times \vec{H} = i\omega\epsilon_0\vec{E} + \vec{J} \tag{2}$$

$$= i\omega\epsilon_0 K \cdot \vec{E}$$

where K is the dielectric coefficient tensor. If the plasma is
characterized by a uniform static magnetic field B_0 parallel to
the z axis of a Cartesian system, K can be written as

$$K = \begin{vmatrix} K_{\perp} & -K_x & 0 \\ K_x & K_{\perp} & 0 \\ 0 & 0 & K_{\parallel} \end{vmatrix} \qquad (3)$$

The components are formed from the basic parameters of the medium, X, Y, and Z.

$$X_s = N_s e^2 / \epsilon_0 m_s \omega^2 \qquad Y_s = |eB_0/m_s \omega|$$

$$\qquad (4)$$

$$Z_s = v_s/\omega \qquad \beta_s = 1 - Z_s$$

where

N_s is the density of sth singly charged components of the plasma.

e is the charge on the electron.

m_s is the ionic mass of the sth component of the plasma.

v_s is the collision frequency of the sth component.

The magnetospheric plasma will be treated as if, at any location, there existed only electrons and one kind of positive ion, which has a mass equal to the average mass of the positive ions at that location. The components of K now become

$$K = 1 - \frac{X_e \beta_e}{\beta_e^2 - Y_e^2} - \frac{X_i \beta_i}{\beta_i^2 - Y_i^2} \qquad (5)$$

$$K_x = - \frac{iY_e X_e}{\beta_e^2 - Y_e^2} - \frac{iY_i X_i}{\beta_i^2 - Y_i^2} \qquad (6)$$

$$K = 1 - \frac{X_e}{\beta_e} - \frac{X_i}{\beta_i} \qquad (7)$$

The subscripts i and e refer to ions and electrons, respectively. If the medium is homogeneous, so that $K_\perp$, K_x, and $K_\parallel$ are independent of coordinates, and if the fields are independent of z (i.e., the wave propagates normal to z), Maxwell's equations can be combined to form the wave equations

$$\nabla_T^2 \vec{E}_z + k_0^2 K_\parallel \vec{E}_z = 0 = \nabla_T^2 \vec{E}_z + k_2^2 \vec{E}_z \tag{8}$$

$$\nabla_T^2 \vec{H}_z + k_0^2 \frac{K_\perp^2 + K_x^2}{K_\perp} \vec{H}_z = 0 = \nabla_T^2 \vec{H}_z + k_1^2 \vec{H}_z \tag{9}$$

where ∇_T^2 is the Laplacian operator in the pair of coordinates transverse to B_0 and k_0 is equal to ω/c.

Cylindrical model. The solution to (9) in cylindrical coordinates is

$$H_z(r,\phi) = \sum_{n=-\infty}^{\infty} [A_n J_n(k_1 r) + B_n Y_n(k_1 r)] e^{in\phi} \tag{10}$$

with J_n, Y_n Bessel functions of the first and second kind and with A_n, B_n arbitrary constants. If A_n and B_n can be evaluated E_r and E_ϕ can be determined from Maxwell's equation. These three field components form the 'fast' mode which can propagate normal to z. The other possible mode comes from the solution to (8)

$$E_z(r,\phi) = \sum_{n=-\infty}^{\infty} [C_n J_n(k_1 r) + D_n Y_n(k_2 r)] e^{in\phi} \tag{11}$$

The two modes are uncoupled, since propagation is normal to B_0.

To describe the fields in a medium that consists of concentric cylindrical layers (each layer being homogeneous but with an axial

static magnetic field), we begin with (10) for one mode and derive from (2) the result

$$E_r = \frac{1}{i\omega\varepsilon_0(K_\perp^2 + K_x^2)} \left[\frac{K_\perp}{r} \frac{\partial H_z}{\partial \phi} - K_x \frac{\partial H_z}{\partial r} \right] \quad (12)$$

$$E_\phi = \frac{1}{i\omega\varepsilon_0(K_\perp^2 + K_x^2)} \left[K_\perp \frac{\partial H_z}{\partial r} + \frac{K_x}{r} \frac{\partial H_z}{\partial \phi} \right] \quad (13)$$

The boundary conditions on each cylindrical boundary, $r = a$, are

$$E_\phi(r = a^-) = E_\phi(r = a^+) \quad (14)$$

$$H_z(r = a^-) = H_z(r = a^+) \quad (15)$$

Because of the form of the solution, these boundary equations hold independently for each n value and have the form, for example of

$$A_n^m J_n(k_1^m a) + B_n^m(k_1^m a) = A_n^{m+1} J_n(k_1^{m+1} a) + B_n^{m+1} Y_n(k_1^{m+1} a) \quad (16)$$

where m is an index specifying the layer in which the field is evaluated.

In the present problem, one boundary will be at the magneto-pause. Exterior to this boundary the field consists of an incident plane wave and a scattered cylindrical wave. The form of the H_z field in the interplanetary region is then, for the fast mode,

$$H_z(r,\phi) = H_0 \sum_{n=-\infty}^{\infty} i^n e^{in\phi} J_n(k_1 r) + \sum_{n=-\infty}^{\infty} F_n e^{in\phi} [J_n(k_1 r) \quad (17)$$

$$+ iY_n(k_1 r)]$$

The innermost boundary to the magnetospheric region is at the surface of the earth. The earth itself is assumed to be a perfect conductor, so that the electric field vanishes for $r \leq R_E$. The boundary condition is then

$$E_\phi(R_E, \phi) = 0 \tag{18}$$

The conditions above (equations 16 and 18) provide just the proper number of conditions to allow a solution to be formed for each undetermined constant in the field expression (equations 10, 12, and 13, for example). In a model of N layers (counting both the earth and the interplanetary medium), the number of unknowns is $2N - 3$. However, because these unknowns are complex there are $4N - 6$ values to be determined, and hence $4N - 6$ real equations. Solution to the set of $4N - 6$ equations determines the coefficients in the field expansions for one n value and one frequency.

Plane model. For propagation normal to the plane boundaries, equation 9 has the solution

$$H_z(x) = H_{z1} e^{ik_1 x} + H_{z2} e^{-ik_1 x} \tag{19}$$

where H_{z1} and H_{z2} are the complex amplitudes of the plane was propagating in the positive and the negative x directions, respectively. The corresponding electric field is

$$E_y(x) = \frac{-k_1 K_\cdot H_{z1}}{\omega \epsilon_0 (K_\perp^2 + K_x^2)} e^{ik_1 x} + \frac{k_1 K_\cdot H_{z2}}{\omega \epsilon_0 (K_\perp^2 + K_x^2)} e^{-ik_1 x} \tag{20}$$

$$E_x(x) = (K_x/K_\perp) E_y(x) \tag{21}$$

The boundary cond'.1ons on the plane boundaries (x is constant)
are that E_y and E_z be continuous. Again, 4N - 6 equations are
solved for the 4N - 6 unknown amplitude values. Only the fast
mode solution is considered.

B. **Specific Magnetospheric Models**

1. **Cylindrical Magnetosphere.** The earth-magnetosphere-interplanetary-
space system is represented in the curved model by a set of twenty-
two concentric cylinders. The innermost, which is the earth, is
a perfect conductor. The next in order of increasing radius is
the neutral nonconducting atmosphere. The following nineteen
cylindrical layers represent the ionosphere magnetosphere. The
last 'layer' is bounded on the inside by the magnetopause, but it
has an infinite outer radius. Each layer is considered to be
homogeneous, with parameters as indicated in Table 1.

The values assigned to the parameters for each layer, together
with the radii appropriate to each layer, were decided as follows.
First, a table of mean molecular weight, electron number density,
and geomagnetic field intensity was compiled for fifty-five
geometric altitudes between 60 and 61,000 km. These values were
taken from the literature [Prince et al., 1964; Matsushita and
Campbell, 1967]. Smooth curves of the three parameters are shown
in Figures 1 and 2. In this model no consideration was given to
collisions of ions with neutrals or with other ions. The consequences
of this neglect will be discussed later. Values were chosen to
represent approximately sunspot minimum, daytime.

The values of the log of the Alfven velocity V_A calculated at
the fifty-five data points were plotted against log R and a smooth

curve was drawn through the points. The ionosphere-magnetosphere
was divided into nineteen regions; their boundaries were chosen
simply by inspection, but in such a way as to fit the V_A curve
as closely as possible, as judged by eye. The set of radii thus
chosen were imposed on smooth curves of K_1, K_x, and values for
each region selected. The number of layers chosen (twenty-two)
was a compromise between an accurate fit and a reasonable amount
of computer time.

2. <u>Plane Magnetosphere</u>. The same parameters were used to describe
the plane layered mode. Each radius listed in Table 1 now becomes
a height x above the center of the plane conducting slab that
represents the earth.

TABLE 1. Parameters of the Twenty-Two-Layer Model

Layer	Radii, meters		V_A, m/sec	K_x (ω = 0.1/sec)	K (α = 0.1/sec)
1	0	6.37×10^6	...	...	...
2	6.37×10^6	6.43×10^6	3.00×10^8	0.00	1.00
3	6.43×10^6	6.47×10^6	2.00×10^6	4.00×10	5.00×10^3
4	6.47×10^6	6.51×10^6	4.40×10^5	5.40×10^2	6.00×10^5
5	6.51×10^6	6.61×10^6	2.90×10^5	1.10×10^3	1.40×10^5
6	6.61×10^6	6.83×10^6	2.30×10^5	1.40×10^3	2.15×10^5
7	6.83×10^6	7.17×10^6	4.00×10^5	5.00×10^2	9.00×10^5
8	7.17×10^6	7.67×10^6	1.00×10^6	6.00×10	1.10×10^5
9	7.67×10^6	8.27×10^6	2.30×10^6	3.00	1.70×10^4
10	8.27×10^6	1.08×10^6	4.80×10^5	4.60×10^{-1}	4.80×10^3
11	1.08×10^7	1.18×10^7	3.85×10^5	1.25	7.50×10^4
12	1.18×10^7	1.34×10^7	3.25×10^6	2.30	1.00×10^4
13	1.34×10^7	1.52×10^7	2.70×10^6	5.30	1.60×10^4
14	1.52×10^7	1.88×10^7	2.05×10^6	1.70×10	2.60×10^4
15	1.88×10^7	2.24×10^7	1.52×10^6	6.00×10	4.60×10^4
16	2.24×10^7	2.54×10^7	1.70×10^6	3.80×10	5.80×10^4
17	2.54×10^7	2.96×10^7	3.50×10^6	2.20×10	5.40×10
18	2.96×10^7	3.44×10^7	2.80×10^6	3.30×10	4.50×10^4
19	3.44×10^7	4.64×10^7	2.85×10^6	1.30×10^2	3.40×10^4
20	4.64×10^7	6.20×10^7	1.50×10^6	9.20×10^2	4.50×10^4
21	6.20×10^7	6.80×10^7	1.45×10^6	1.25×10^3	5.00×10^4
22	6.80×10^7	∞	2.12×10^5	4.31×10^5	2.58×10^6

Layer 1 represents the earth; layer 2, neutral atmosphere; layers 3-21, the ionosphere-magnetosphere; layer 22, interplanetary space.

C. **Results of Computations.**

1. **Calculations of Magnetic Field Amplitude Transmission Coefficient for First Two Cases.** The cylindrical model leads to a set of eighty-two simultaneous real linear algebraic equations for the coefficients A_n^m, B_n^m, and F_n in the field expressions (equations 10 and 17); the superscript designates the nth layer. One value of u was chosen, and the set of equations was solved for each integer n in the interval $-10 \leq n \leq +10$. The values for A_n^2 and B_n^2 were then substituted into (10), and the magnitude H_z from 2 to 100. The trend of the ratio to increase with increasing frequency is apparent. The resonant frequencies for both cases fell within the range of continuous sinusoidal oscillations observed at geostationary altitudes (0.02 to 0.12 radian/sec) and interpreted as magnetospheric resonance standing waves [Cummings et al., 1969].

The low frequency limits for the two cases differ by a factor of 2. This result could have been anticipated, since at these frequencies the magnetosphere has little effect on the wave. Thus, as $\omega \rightarrow 0$, the comparison is between the reflection at an infinite conducting plane and the scattering by a conducting cylinder. The fields calculated in these cases exhibit a factor of 2 in the ratio of the amplitudes of the magnetic intensity at the surfaces.

Field and Greifinger [1966] defined their 'ionospheric magnetic transmission coefficient,' t_B, as the ratio of the total magnetic field at the base of the ionosphere to the field of the downcoming

part of the signal at the top of the ionosphere. The top of the
ionosphere was taken to be 450 km above the surface for daytime,
sunspot maximum conditions; the base of the ionosphere, to be
85 km high. The model ionosphere of which they make use has a
constant Alvén speed in this region, $V_A = 200$ km/sec, a constant
ion cyclotron frequency of 100 radians/sec, and ion and electron
collision frequencies proportional to each other and both exponential
functions of height. Obviously the present model is not identical
to that of Field and Greifinger, even aside from collision effects.
Nevertheless, a comparison of their computed t_B with an ionospheric
magnetic transmission coefficient calculated on the basis of the
present plane layered model should give some indications of the
attenuation resulting from collisions. Such a comparison is
illustrated in Figure 4, where t_B' is the corresponding transmission
coefficient derived from the present work. The evaluation of t_B'
was made by comparing the amplitude of the magnetic field at the
surface with the amplitude of an incoming plane wave above the
boundary at 6.83 x 10 meters, or 460 km above the surface. Ionospheric
reflections of the incident wave causes the gradual decrease in t_B'
as the frequency is increased, until the layered medium becomes
resonant at above 6 radians/sec. The t_B curve decreases rapidly
because of absorption, and so shows no resonance. The ratio of the
two curves t_B/t_B' is shown in the broken curve in Figure 4.

To make use of the Field and Greifinger result, the spectra
of the plane and cylindrical magnetospheres were multiplied by the
ratio t_B'/t_B. The application of this quantity, based on a one-
dimensional calculation, to the cylindrical model seems justified

since the thickness of the ionospheric layers involved is small
in comparison with the radius of curvature of the bounding
surfaces, namely 460 km/6370 km, or about 7%. The final spectra
including the collision effects are shown in Figure 5. The
ratio of these two curves, P for the plane layered model and
C for the cylindrical one, is of course still the same as in
Figure 3.

One difference in the two transmission curves, in addition to
the depressed transmission in the cylindrical case brought about
by scattering, is the shift in resonant frequencies. The first
resonance comes at a higher frequency in the cylindrical model
than in the plane mode. Examination of the field amplitude through-
out the medium indicates that this effect is a result of multiple
internal reflections in the plane model, which lead to a longer
effective path length for the wave propagating normal to the
plane bounding surfaces. The higher resonant frequencies are
harmonically related to the first resonance in the cylindrical
model, but their amplitudes are rapidly decreasing, owing to the
scattering at each curved boundary. The relationship between
resonant frequencies is more complicated in the plane model because
of multiple reflection and individual layer resonance occurring
at frequencies above 0.35 radian/sec.

The sensitivity of the results to the details of the models was
investigated by varying the magnetospheric parameters over a rather
wide range of values. In general, the spectra are only slightly
sensitive to such changes. To illustrate this insensitivity, a

calculation was made for both models in which the entire magneto-
sphere ionosphere was replaced by a single homogeneous, isotropic
layer. The dielectric constant of this layer was adjusted to
give the same propagation time as in the detailed models. The
result in the cylindrical case is a spectrum displaying resonances
that coincide with the first few resonances in the more elaborate
model but having a transmission coefficient that falls off more
slowly with frequency. The fact that the transmission coefficient
is not strongly coupled to the specific details of the magnetospheric
model has two implications: First, the approximations used in the
claculations for the cylindrical model tend to be justified; second,
the transmission coefficient for the magnetic field amplitude is
seen to be much more strongly dependent on the geometry of the magneto-
spheric model than on, for example, the details of parameter profiles.

2. **Progress in Two-dimensional Dipole Case.** A "two-dimensional
dipole" field was assumed:

$$\hat{B}(r,\phi) = \frac{\mu_o M}{\pi r^2} \ (- \sin \phi \ \hat{r} + \cos \phi \ \hat{\phi})$$

The wave equations for the wave field components are strongly
coupled by this static field structure. A method of decoupling
lccally was found, which in principal would allow solution to the
wave equations in a small volume. However, the boundaries for
these small volumes were such that the boundary condition could
not easily be ratified. A compromise static field structure was
next employed. This ambient field was radical at high "latitudes"
($\frac{\pi}{4} \le \rho \le \frac{3\pi}{4}$) and was azimuthal at low latitudes ($-\frac{\pi}{4} \le \rho \le \frac{\pi}{4}$).

This static field has some characteristics of the dipole field,
but again boundary conditions could not be solved. Finally, it
was found that in a thin cylindrical shell (of thickness t where
t << r, and r = inner radius of the shell) the wave equation
could be decoupled. For this case the wave equation and the
dispersion relation can be solved numerically, and the boundary
conditions satisfied. However, numerical calculations are not
complete. Thus, the evaluation of the propagation of h-u waves
through a "two dimensional dipole" field in a cylindrical magneto-
sphere was weakened to the determination of propagation through a
thin cylindrical shell of plasma with an azimuthal field. This
problem, while soluble, was not completed in the contracting
period.

III. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

A. Summary and Conclusions. A comparison of the magnetic field
amplitude transmission coefficient for two model magnetospheres,
one of cylindrical geometry and the other plane geometry, reveal
a significant difference. The curved magnetosphere scatters
and diffracts an incident wave, and so transmits less energy to
the surface. The ratio of the transmission coefficients in the
two cases varies from 2 to about 100 over the hydromagnetic range
of frequencies. The lowest resonant frequencies are seen to depend
largely on the size of the magnetosphere and average velocity of
hydromagnetic waves. However, a plane layered model may produce
more complicated resonant behavio. because of multiple internal
reflections. A fault of a one-dimensional treatment is its over-
emphasis of the effect of the earth on magnetospheric propagation,

a fault inherent in the plane mode. Previous calculations that were based on a model involving plane layers and that indicated amplification effects at discrete frequencies should be reexamined if comparison between interplanetary field and surface field amplitudes is desired.

The evaluation of propagation characteristic in a magnetosphere with curved static field was attempted, but not completed. Such a calculation would suggest possible lati udinal variation in surface wave field intensities due to the guiding tendency of the ambient field.

B. Recommendations. It is recommended that additional effort be put into the curved field calculations. Scattering, diffracting, and guiding effects in this case should make the propagation characteristics differ considerably from the two previous calculations, and should provide an indication of the latitutional variations in surface field intensities to be expected from h-m waves propagated through the magnetosphere.

IV. PERSONNEL

The personnel employed under this contract were Dr. A. W. Jenkins, Jr., Mr. Hector Dinas, and Mr. William Lee. Mr. Dinas and Mr. Lee were employed as graduate research assistants. Dr. Jenkins was principal investigator.

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VI. PUBLICATIONS

Two Publications resulted from this contract:

1. A. W. Jenkins, "A Comparison of Hydromagnetic Wave Propagation
   in Magnetospheres of Pland and Cylindrical Geometries", J.
   Geophys. Res., 75, 3817-3824, 1970

2. Hector Dimas, "Propagation of Hydromagnetic Waves in a Cylindrical
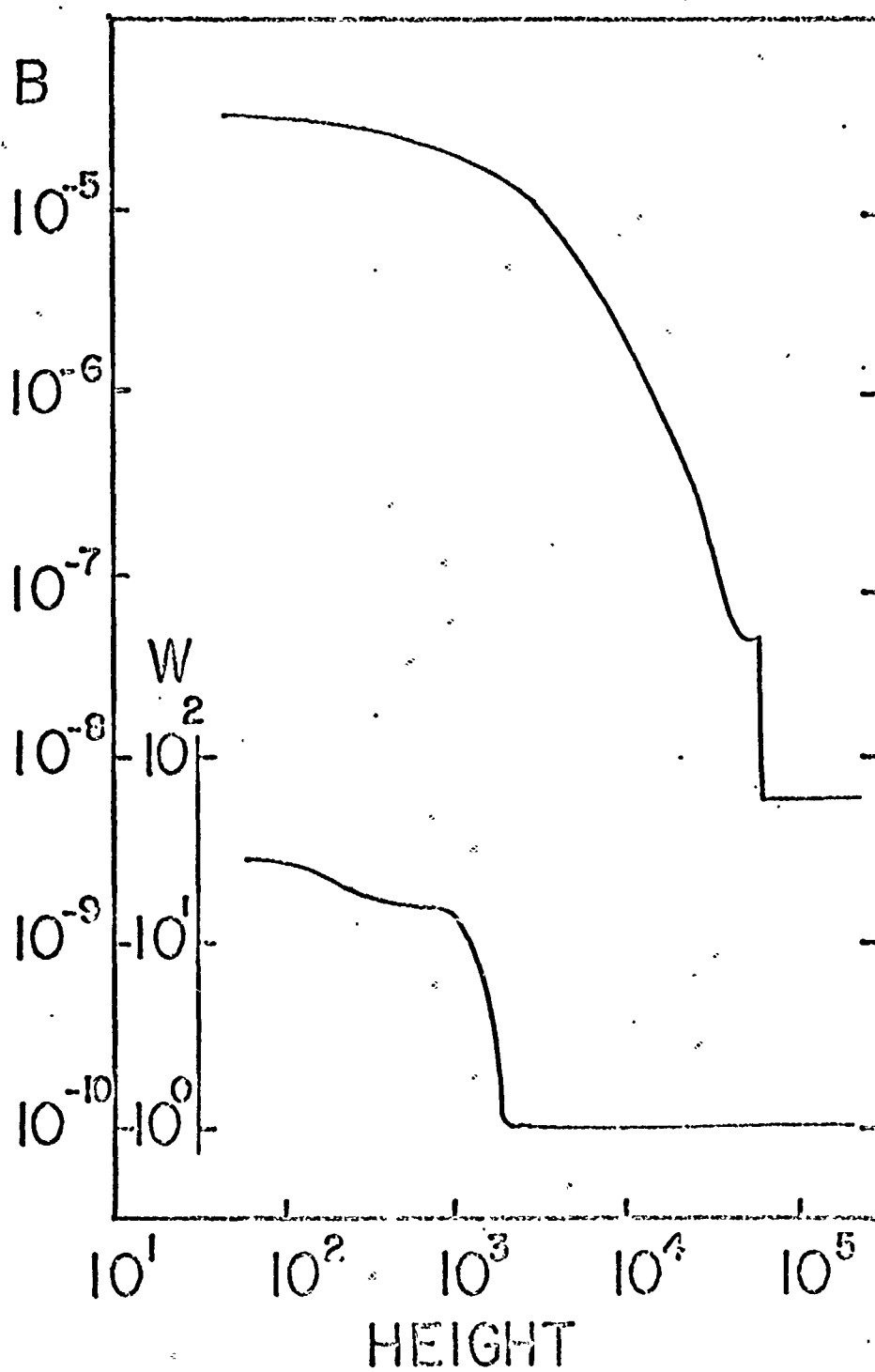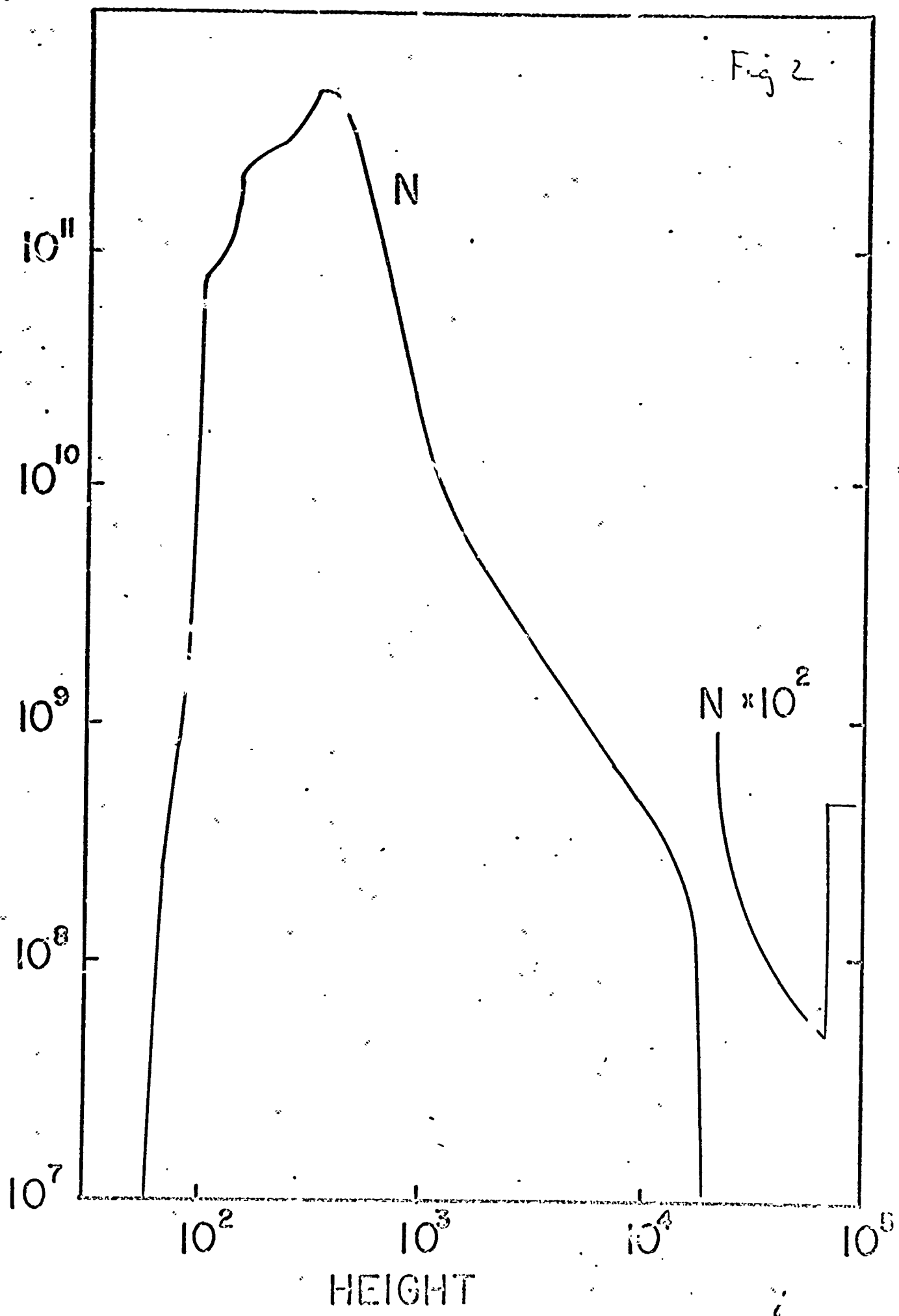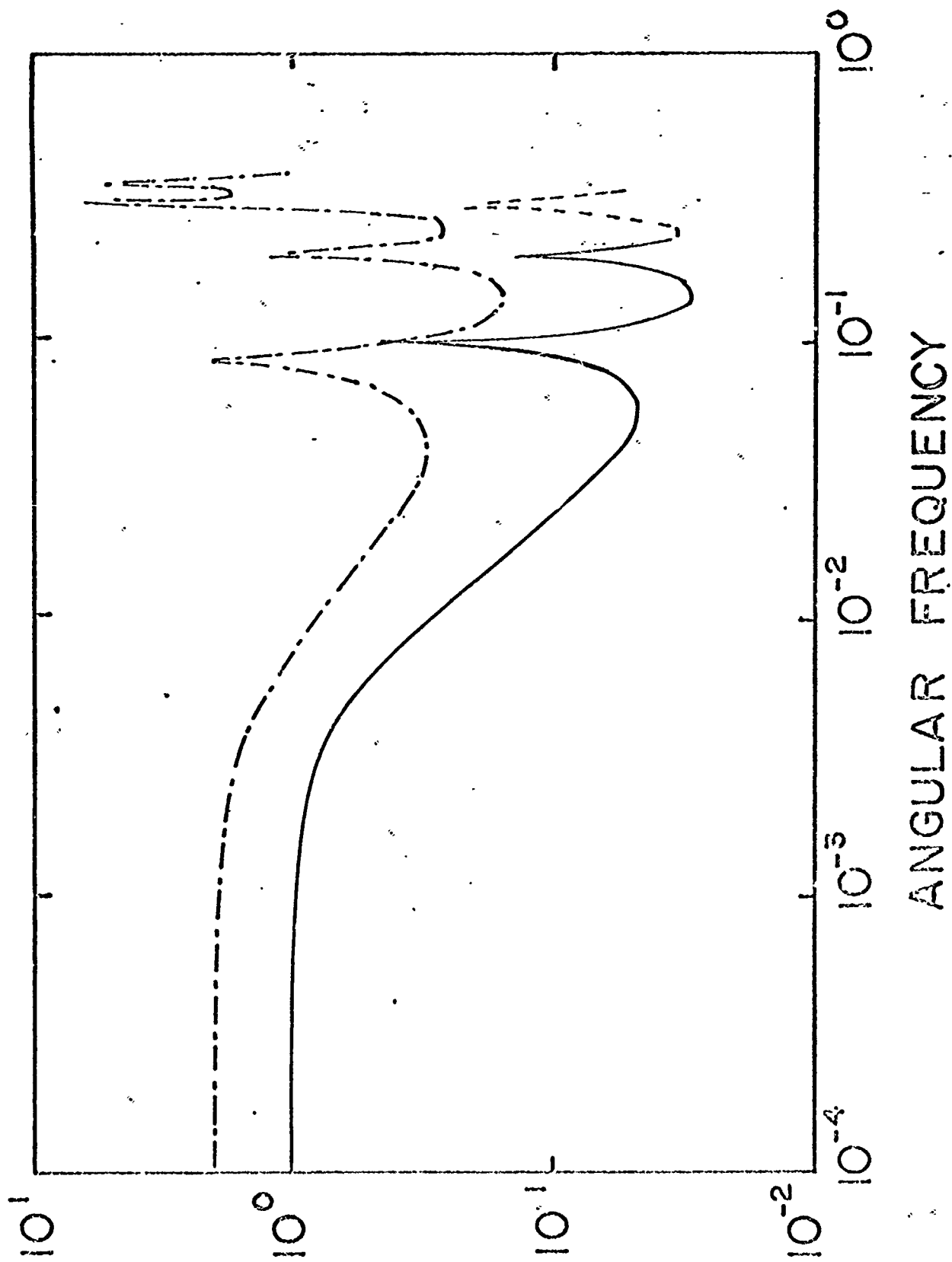   Magnetosphere," Thesis, N. C. State University, 1969.

Fig 1

Fig 2

N

N ×10²

HEIGHT

ANGULAR FREQUENCY

Fig 4